

## Heckel Equation

$$\ln E = -(kP + a)$$

E is a Function of Compact Porosity

$$E = 1 - D' / D$$

D' Apparent Density (Compact Weight/  
Compact Volume)

D True Density ( $\text{g/cm}^3$ )

## Heckel Equation

$$\ln E = -(kP + a)$$

- Applied Pressure Increases
- Apparent Density Increases to a Limit
- Total Porosity Is Determined as a Function of Pressure and True Density

# Heckel Equation

Commonly in Working Forms

$$D_{rel.} = D'/D$$

## Heckel Equation

Most Widely Used Form:

$$\ln \left( \frac{1}{1 - D_{rel.}} \right) = kP + a$$

P Applied Pressure

k Is a Constant Relating to Yield Pressure

a Related to Original Compact Volume

## **Heckel Equation**

**k Is a Constant Equal to Reciprocal of the Mean Yield Pressure**

$$k = 1/P_y$$

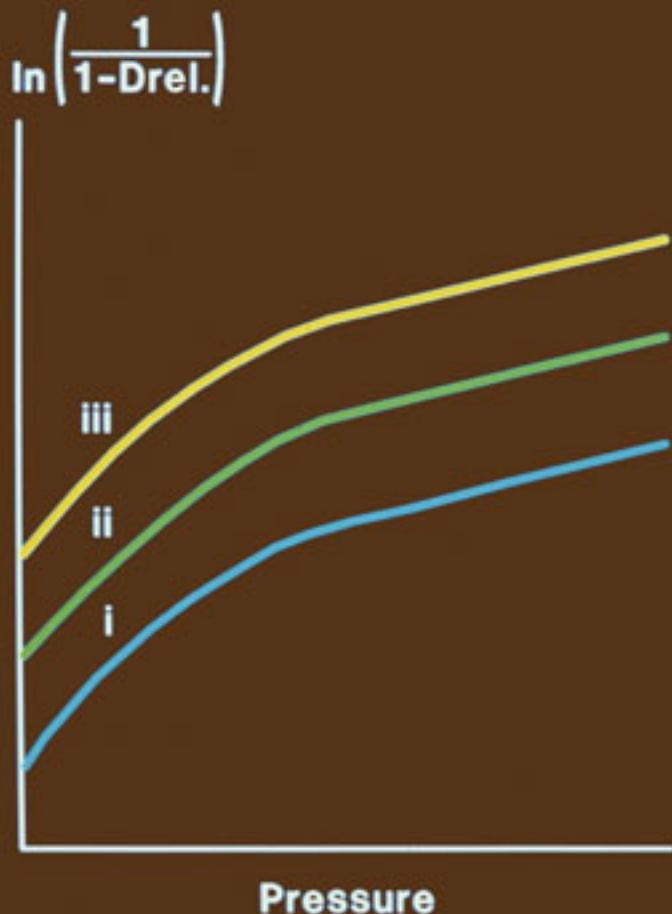
**Obtained from Linear Portion of a Plot**

$$\text{of } \ln\left(\frac{1}{1-D_{rel.}}\right) \text{ vs. } P$$

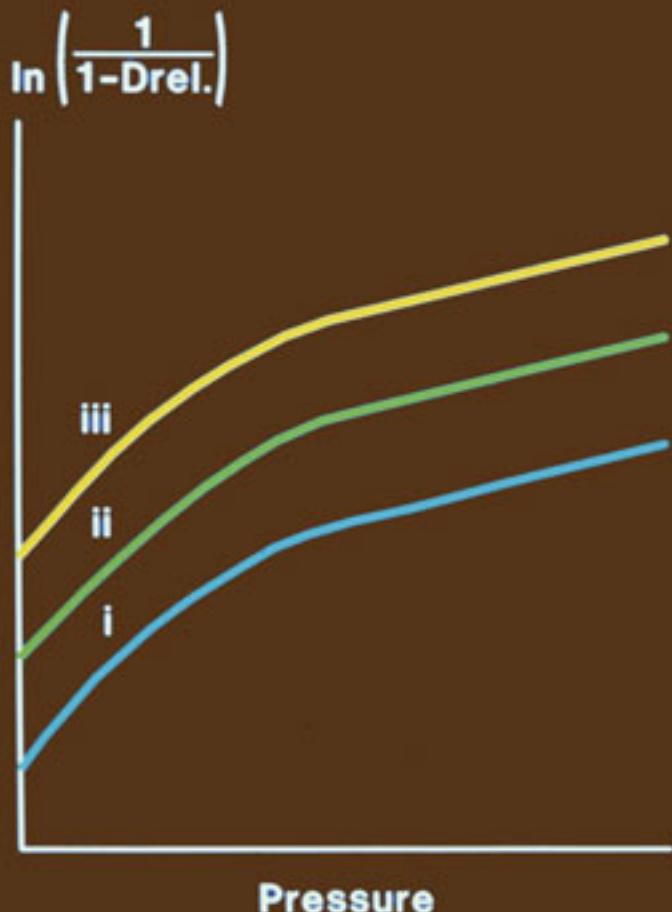
# Heckel Plots

- Working Equation Is Plotted  
 $\ln \left( \frac{1}{1-D_{rel.}} \right)$  vs. P
- Plots May Be Broken Down into 3 Types
- Only Two Types Will Be Discussed
- Type A, Characteristic of Soft Materials, Plastic Deformation
- Type B, Characteristic of Hard Materials, Brittle Fracture

# Heckel Plot Type A

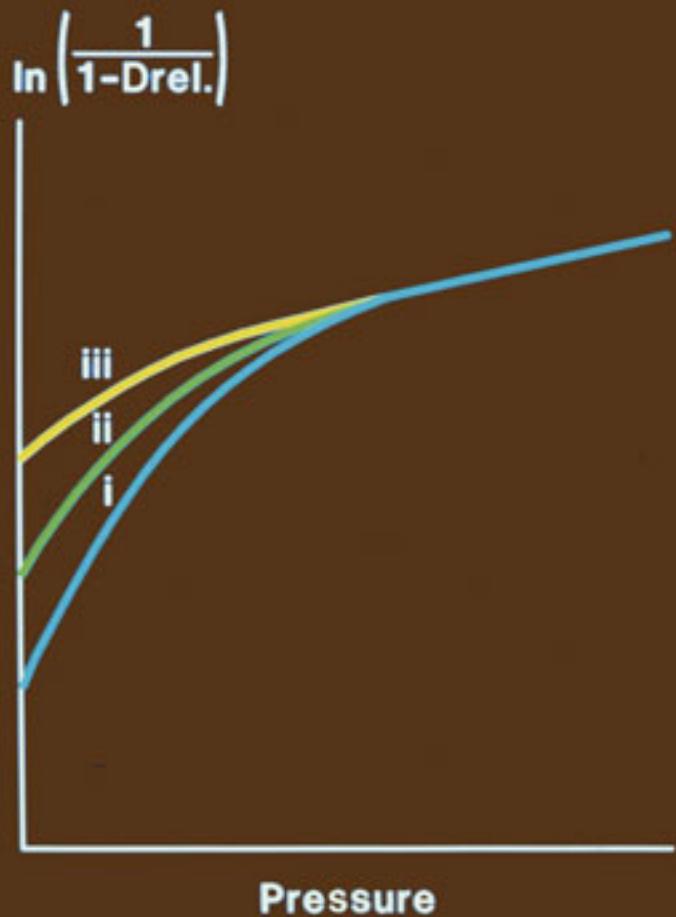


## Heckel Plot Type A

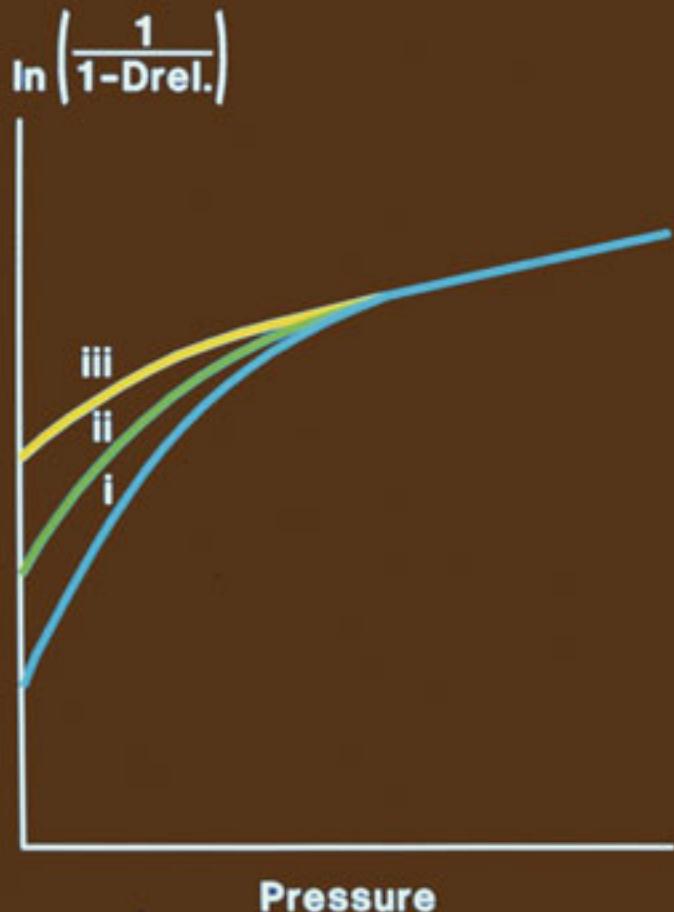


- Densification Proceeds, Slip and Rearrangement
- Plastic Flow Occurs Beyond the Yield Point
- Dependent on Initial Particle Size and Density
- Finished Compacts Retain Different Degrees of Porosity

## Heckel Plot Type B



## Heckel Plot Type B



- Densification Proceeds, Particle Fragmentation
- Fragmentation Yields a Compact of Consistent Porosity
- Convergence to a Common Line, Independent of Particle Size

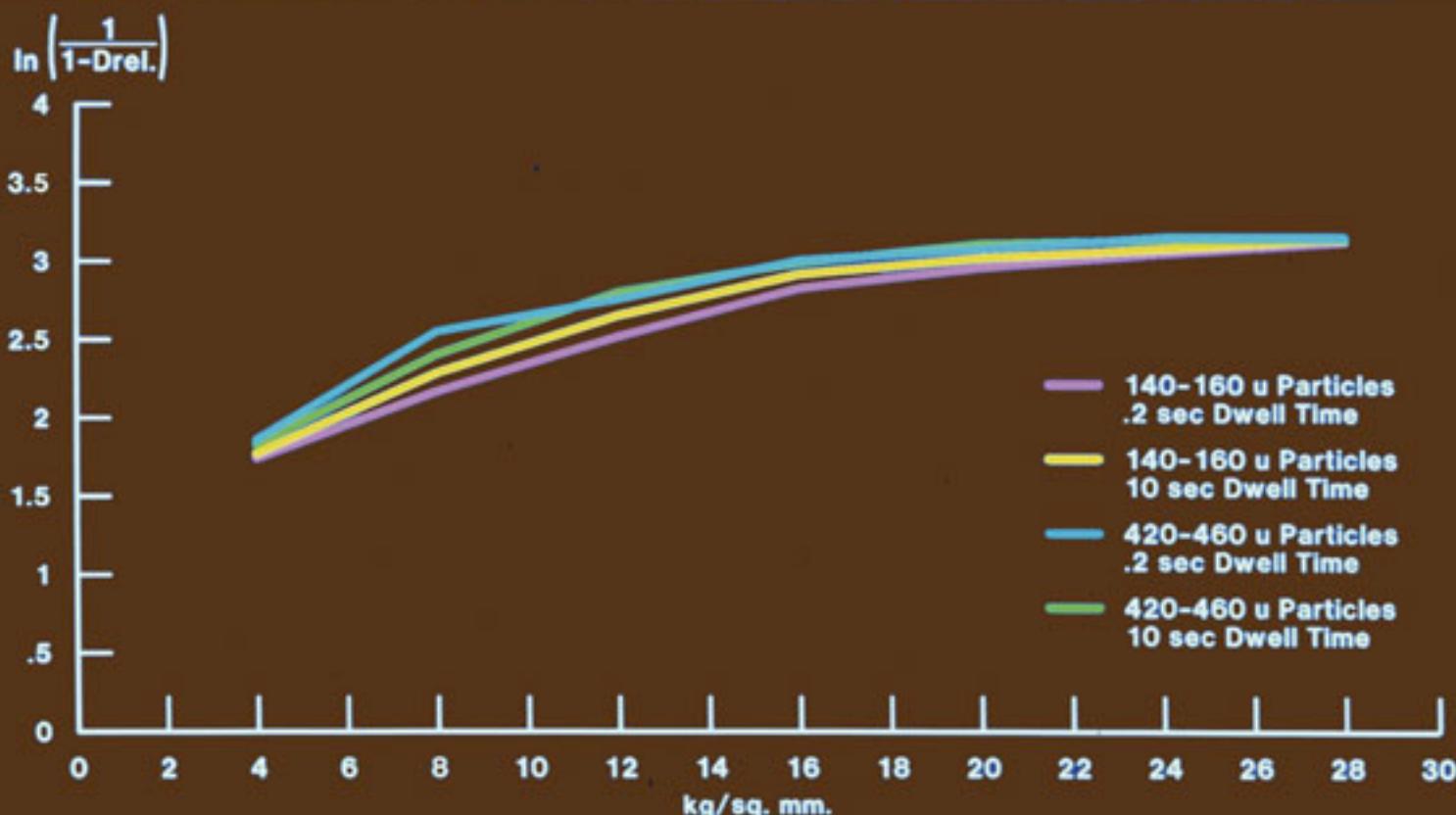
## **Heckel Equation Special Considerations**

- **Make Compact Measurements at Zero Pressure**
- **Permits Compact to Recover Elastically**
- **Density Changes Are the Result of Non-Recoverable Deformation/Fragmentation**
- **Measurements under Pressure Include an Elastic Component Which Increases the Value In  $\ln \left( \frac{1}{1-D_{rel.}} \right)$**

## **Heckel Equation Special Considerations**

- Use Two Dwell Times (Contact Times)
- Generate Two Curves
- Difference in the A.U.C. Is the Degree of Deformation

# Heckel Plot Showing Differences in the A.U.C.



# **Heckel Equation Variation of Methodology**

- **Detailed Knowledge of Initial Packing Is Not Needed**
- **It Has Proven to Be Relatively Insensitive to This Parameter**
- **Confirmed by Researchers Varying Bulk Density**

# **Compression Cycles**

## **Cycle Plots**

- **Diagrammatic Representations  
of Radial Force Plotted as a  
Function of Axial Force**

# Cycle Plots

Axial Force, Pressure Supplied  
by Punches

Radial Force, Pressure Transmitted  
Perpendicular to the Axis of the Punch

# Cycle Plots Types

Perfectly Elastic

Constant Yield Stress during Shear

Mohr Body

# **Cycle Plots Mechanisms**

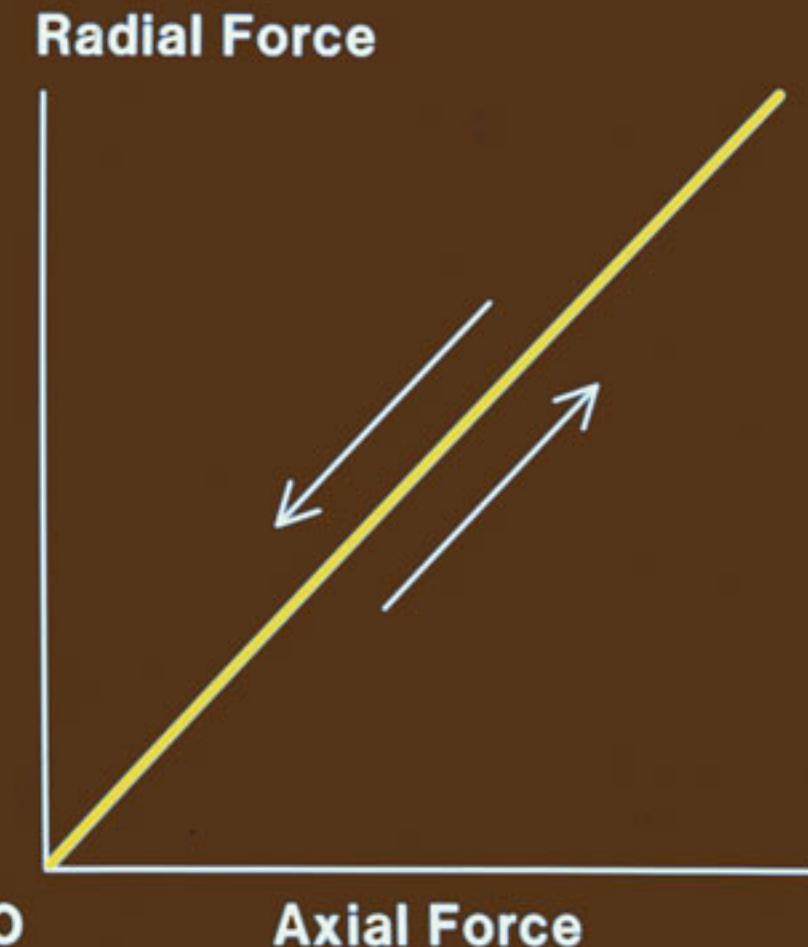
**Perfectly Elastic, Rarely Seen  
Constant Yield Stress,  
Plastic Deformation  
Mohr Body, Brittle Fracture**

# Cycle Plots Perfectly Elastic Body

- Axial Force Is Applied
- Radial Force Is of Equal Magnitude
- Poisson Ratio

$$\frac{\text{Axial}}{\text{Radial}} = \frac{\sigma}{\tau} = \nu$$

# Perfectly Elastic Body



# Cycle Plot Perfectly Elastic Body

**Characteristically as Axial Force  
Is Decreased, Radial Force  
Decreases along Same Line, No  
Residual Forces Are Exerted  
on Die Wall**

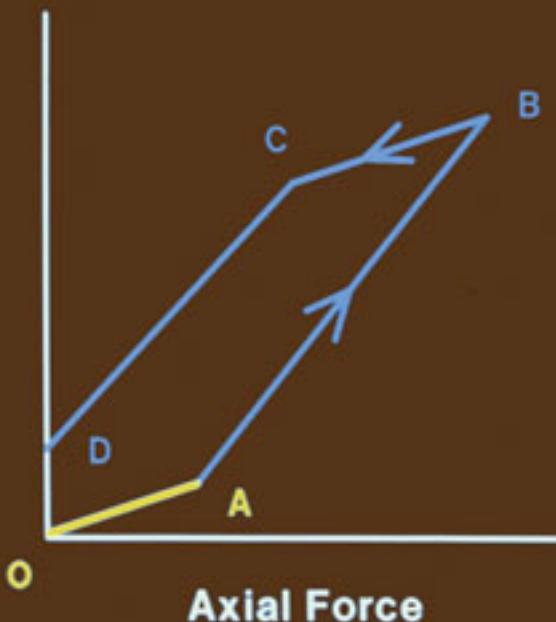
# **Cycle Plot Constant Yield Stress During Shear**

- **Characteristic of More Conventional Tablet Materials**
- **Diagram Is Somewhat More Involved Due to Residual Forces, Relating to Viscous Behavior**

## Cycle Plot Constant Yield Stress During Shear

Line O-A, Elastic Region Below Yield Point  
Beyond A Deformation Begins

Radial Force



# Cycle Plot Constant Yield Stress During Shear

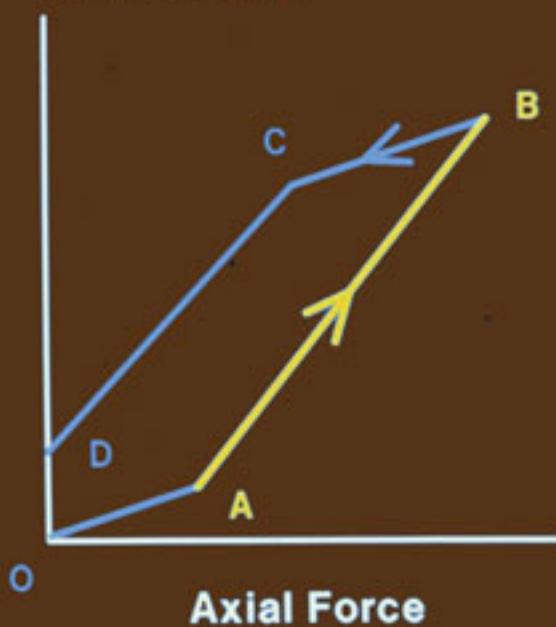
**Yield Stress Is a Function of Radial and Axial Factors**

$$\text{Axial } (\sigma) - \text{Radial } \tau = 2S$$

**S Represents Yield Stress**

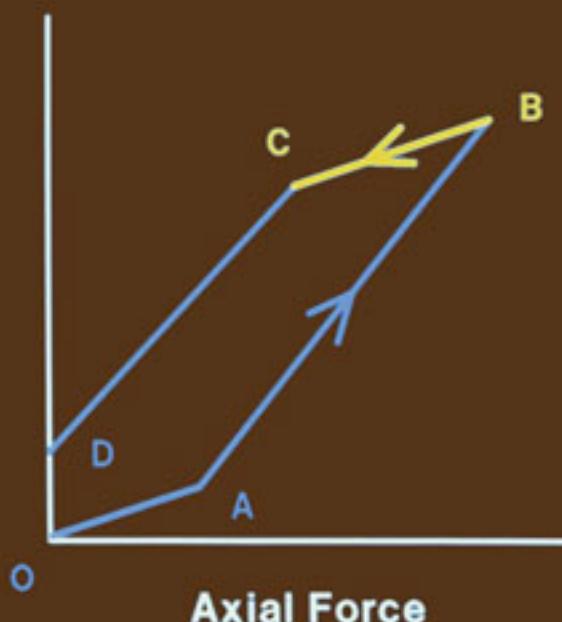
## Cycle Plot Constant Yield Stress During Shear

Line A-B Yield Stress is Constant  
Point B, Axial is Decreased, Yield under Shear Stops  
Radial Force



## Cycle Plot Constant Yield Stress During Shear

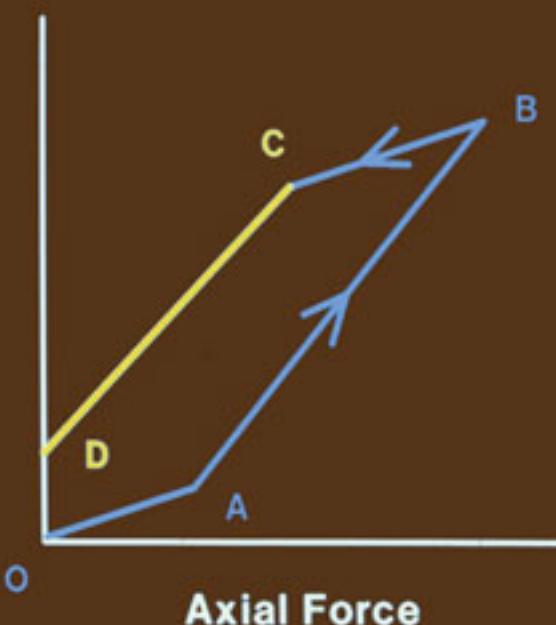
Line B-C Die Wall Force Decay  
Point C if Force Is High Enough, Yield Point to Radial Force  
Radial Force



## Cycle Plot Constant Yield Stress During Shear

Line C-D Same Relationship as A-B  
Point D, Amount of Force Exerted on Die Wall

Radial Force



# Cycle Plot Mohr Body

This Term Refers to the Mohr - Coulomb Yield Stress at the Plane of Shear for Two Bodies

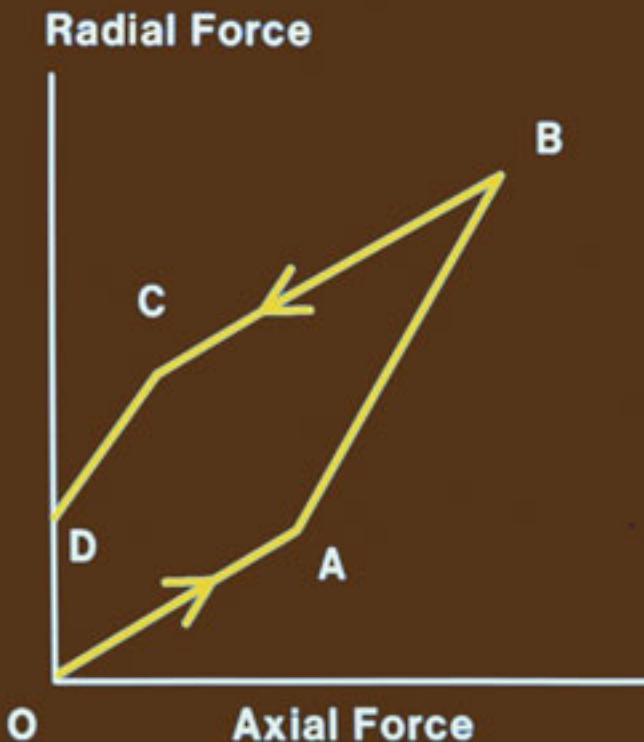
## Cycle Plot, Mohr Body

**Mohr - Coulomb Relationship,  
Using Yield Stress, S**

$$\tau_n = S + \mu \sigma_n$$

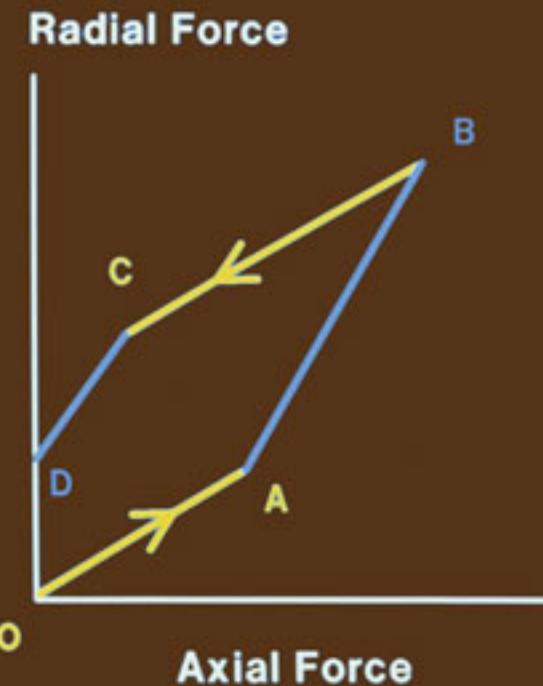
**$\tau_n$  Shear Stress on Slippage Plane**  
 **$\mu$  Frictional Coefficient**  
 **$\sigma_n$  Normal Stress**

# Cycle Plot, Mohr Body



# Cycle Plot, Mohr Body

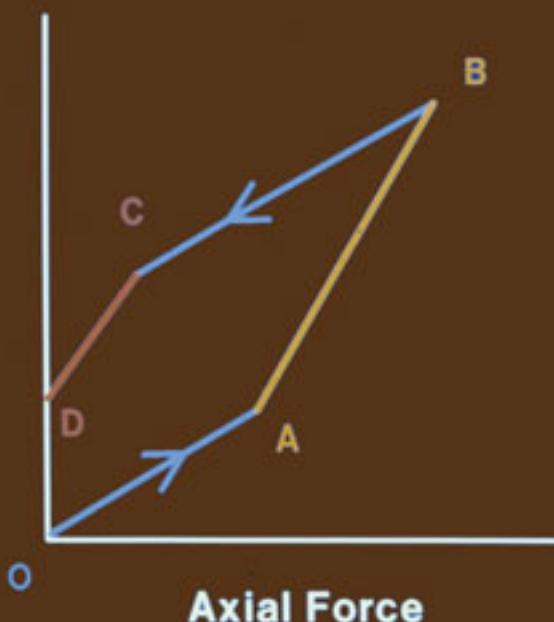
Force Must be Large Enough to Detect Points A,C  
Slopes OA = BC =  $\sqrt{\nu}$  Poisson Ratio



# Cycle Plot, Mohr Body

Slopes CD and AB Are Different,  
Suggest Mohr Body

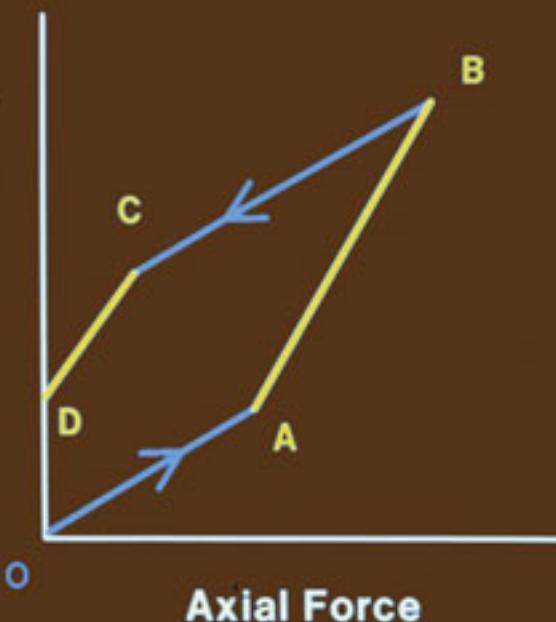
Radial Force



# Cycle Plot, Mohr Body

Slopes CD and AB Become Equal,  
Constant Yield Stress

Radial Force



# Cycle Plot Special Considerations

- Most Mathematical Models Assume Ideality, Nonporous
- Real Systems Are Not Isotropic in Structure
- Strict Assignment of Mechanisms May Not Be Desirable
- Quantitative Interpretation Should Give Way to More "Ad Hoc" Usage

# Cycle Plot, Constant Yield Stress

